DOCUMENT RESUME

ED 334 227 TM 016 724

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TITLE Features of Knowledge Representations for Data Presented Graphically in Populations with Diverse Mathematical Experience.
PUB DATE 90
NOTE 19p.
PUB TYPE Reports - Research/Technical (143)

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS *Cognitive Processes; *College Students; Comparative Analysis; Concept Formation; *Experience; *Graphs; Higher Education; Knowledge Level; *Mathematics; Memory; Perception; Statistics; Student Attitudes
IDENTIFIERS *Knowledge Representation; *Mental Models

ABSTRACT

Knowledge representations and mental models from graphically presented information were investigated for 98 college students with different levels of mathematics experience. The subjects were drawn from a remedial algebra course (n=35), a second semester calculus course (n=35), and a calculus-based statistics course (n=28); they were all exposed to 40 graphical stimuli of bar graph variations. Subjects were asked to judge the typicality and readability of each stimulus. To judge these qualities, subjects required a working model of graph comprehension and a model of concept formation and structure. Results confirm that both memory processes and perceptual processes affect the views of subjects concerning how easy or hard each graph was to read and understand. Individuals of different mathematics experience had different views of the typicality of bar graph stimuli, with increasing complexity of conceptual structure with increasing experience. Nine figures present study data. A 17-item list of references is included. (SLD)

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Features of Knowledge Representations for Data Presented Graphically in Populations with Diverse Mathematical Experience

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The ubiquity of computing power, word processing, presentation graphics packages and desktop publishing has resulted in the inundation of students and other citizens with data presented graphically for informative, persuasive or at times merely cosmetic purposes. This change in the working information environment of adults and students has put a premium on the critical interpretation of graphically presented data. The skills needed for such critical interpretation require a strong research base in the perception, representation and utilization of knowledge embedded in graphically represented information. For instance, it is essential to understand the typical cognitive processing of any type of information to design instruction effective to enhance that cognitive processing. Such is certainly the case for critical evaluation of quantitative information presented graphically. An important question is what are the cognitive processes central to the comprehension and use of graphical stimuli in question answering and other practical tests.

Considerable work has been done on interpreting graphs, beginning over fifty years ago (e.g., Washburne, 1927; Vernon, 1946, 1950; Peterson & Schramm, 1954; MacDonald-Ross, 1977). More recently, attention has been given more broadly to the questions of the visual display of quantitative information (Tufte, 1983), although not in a research context. Some work has been done on mathematical graphs as they relate to the idea of mathematical functions (e.g., Leinhardt, Zaslavsky & Stein, 1990). Work has also begun on the critical evaluation of quantitative arguments in more and less expert subjects (McKnight, 1990; McKnight, Kallman, & Fisher, 1990). Some research attention has been given recently to the perception of graphical data (e.g., Peterson & Schramm, 1954; Cleveland, 1985). Research attention now seems to be turning to dynamic and interactive graphics, that is, cognitive processing and learning related to graphics-based presentations in highly interactive computer formats.

While the above research is important and the turn toward research in understanding graphics in the interactive environment of the microcomputer, the turn from attempting to understand and model the cognitive processing related to use of static, traditional presentation graphics in print media may be premature and should likely supplement the other research. Little research has yet been done to date on the knowledge representations and mental models which result from the perception of graphical information. Since an understanding of knowledge representation is an essential connecting link between research on graphical perception and research on reasoning and question-answering (based on assumed mental models and knowledge representations of various types), that research is both important as a research topic and as a basis for improved interventions in an area important to an educated citizenry in today's society.

Theoretical Framework

The present study presents results from one investigation of knowledge representations stemming from interaction with one kind of graphically presented information (bar graphs of various types) for college students with three differing levels of mathematical experience. Essentially in this experiment, subjects were

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exposed to a set of 40 graphical stimuli covering a variety of bar graph variations (judged so a *priori* by the experimenters) from straightforward simple bar graphs to more complicated versions described below. Subjects were asked to judge the typicality and readability of each stimuli. To interpret the significance of the typicality effects and readability effects found requires two pieces of theoretical background: (1) some working model of graph comprehension, and (2) some model of concept formation and structure during incidental learning (i.e., when formation of the concept involved was not the direct target of formal instruction).

Cleveland (1985) developed some elements of a model of graph perception. Kosslyn (1985, 1989) developed some elements of a model of memory's role is graph comprehension. Pinker (1981) developed a model of graph comprehension tasks that has been the basis of much of the research on understanding graphs. Figure 1 represents a modified version of the key elements and processes of Pinker's model combined with elements from Kosslyn.

Insert Figure 1 about here

Essential to the model portrayed in Figure 1 are the following elements:

1. External stimuli to activate cognitive processing
   a. A *graphical stimulus*
   b. A *problem stimulus* involving an essential reference to the graphical stimulus and, possibly, other contextual information.
2. At least three elements of human information processing "hardware":
   a. A *sensory register* in which the results of perception are accumulated, the activation of the appropriate elements of long-term memory is begun, and an at least partially integrated representation is activated in working memory.
   b. A *long-term memory store* in which contextual, problem-solving and graph interpretation relevant information is stored for activation both by the contents of the sensory register and/or the contents of working memory.
   c. Some form of shorter-term *working memory* in which the contents of the sensory register and long-term memory are integrated into a representation of the current graphical stimulus and comprehension task. In turn, their representations and task elements activate both (1) further demands for information from long-term memory and the sensory register and (2) cognitive processes that move from graphical and task representation to a desired information goal, an answer.

The key processes in this model are (1) *attending* (observing the graphical stimulus and forming an icon in the sensory register; (2) *categorizing* (accessing an appropriate mental model or schema for the graphical stimulus and problem situation); (3) *activating* the appropriate schema (placing the model accessed during categorization into working memory); and (4) *integrating* (guiding further attention and perception to pull information from the graphical stimulus and fit this information to the schema or mental model accessed from long-term memory to serve as the core of the representation needed for cognitive processing to accomplish the task). While the separation of the model elements and processes for discussion may suggest one-way and one-time flow of processing from element to element, the model is actually assumed to be interactive and continuously interactive.

The processes from the modified Pinker model can be broken down into two types of processes: perceptual and memory processes. Cleveland (1985) has worked extensively with the nature of graphical perception involved in understanding graphical stimuli. He has isolated ten elementary graphical perception processes — angle, area, color hue, color saturation, density (amount of black), length (distance), position along a

common scale, position along identical nonaligned scales, slope and volume. To this might also be added that of inferring length from repeated symbols as in an isotype graph. Work on the role of memory processes on graph comprehension has been addressed by Kosslyn (1985, 1989). He has looked at how restrictions in the three types of human memory influence comprehension of graphical stimuli.

Pinker's model assumes some form of graph schema — stereotypical mental models of graphical stimuli residing in long-term memory — to play a central role in graphical comprehension. Memory research, and in particular the literature on concept formation and structure, has proposed a complex array of possible cognitive representations of knowledge and information. Beginning a research program in critical evaluation of quantitative arguments (cf. McKnight, 1990; McKnight, Kallman, & Fisher, 1990) soon made it clear that more insight into the nature of memory representations of graphically-presented data is essential to further understanding of the processes of graphical comprehension and understanding, which, in turn, underlies the development of effective instructional interventions to increase quantitative literacy.

Clearly a very diverse literature exists on knowledge representations, mental models, etc., and on concept formation. While more extensive and systematic investigation of knowledge representations related to graphically presented data are planned for the future, the experiment discussed here uses as its theoretical and interpretive framework the idea of mental models and mental prototypes put forward by Rosch some time ago (1973, 1976, 1978).

This assumption of exemplar or prototype models of concept formation is to some extent problematic for two reasons. First, it is not clear to what extent the graphical stimuli chosen represent coherent concepts in the mental representations of subjects and not simply the a *priori* definition of a category that exists only in the minds of the experimenters. Second, even granting that coherent concepts of the graphical stimuli were formed in subjects' minds around prototypes or exemplars, both prototype and exemplar models of concept formation and structure have recently been criticized (e.g., Medin, 1989) as inadequate. While the niceties of this discussion of concept structure are worth exploring, all that seems necessary for the present initial investigation is a model of concept formation that (1) allows for concept formation over time through incidental learning, and (2) views concepts as "fuzzy," as probability distributions of category inclusion judgements for a range of related stimuli. While 'bar graphs' are certainly abstract and symbolic constructs, the inclusion of a number of "borderline cases" among the stimuli of the experiment makes the category more closely resemble natural, semantic categories ('birds', 'tools') rather than artificial concepts of categories constructed for laboratory experiments.

In particular, the framework assumes that (1) distributed exposure to stimuli of a broadly similar kind leads to implicit mental models (a probability distribution of classifications, whether or not linked to exemplars or prototypes of objects in that stimuli category), (2) that such prototypical mental representations are linked to verbal labels for the stimulus category, and (3) that these prototypical models have features which may be used to characterize them and may serve as the bases for measuring the impact of interventions designed to change those more informally developed mental models. Since, by the first assumption, the prototypical models are formed by "distributed exposure" to stimuli of a certain category, it seems reasonable to extend the framework to suggest that individuals in groups of subjects differing in the amount of exposure to the relevant stimuli are likely to form differing mental representations of the concept linked to the amount and quality of stimulus exposure and that, if the groups are sufficiently homogeneous in experience, the representations for differing groups will reveal features on which the groups as aggregates differ significantly.

In the case of the present experiment, the stimulus category is that of "bar graphs" to which individuals are differentially exposed both from the text materials of schooling and from information in popular media.

It is assumed that, as Rosch suggests for other cases, individuals will build up incidentally to their work with bar graphs in the contexts of schooling and media, implicit mental representations of what bar graphs are like. Based on the experience that led to the building of these representations, there are likely to be differing perceptions of the various kinds of bar graph stimuli — in particular, how typical each type is within the category of bar graphs and how easy each type is to interpret. Further, while formal educational experiences in mathematics and statistics may deal explicitly with bar graphs only rarely, it seems likely that there are sufficient interactions with these stimuli in the context of such experiences that individuals with more mathematical experience may have different conceptions of what is typical and easy to interpret than do individuals with less mathematical experience.

The Experiment

An instrument dealing with various kinds of bar graphs was administered to 98 college students from classes of three different mathematics courses — students in a remedial algebra course (35 subjects), second semester calculus (35), and a calculus-based introductory course in applied statistics typically taught to seniors and beginning graduate students (28). Each class was assumed to be relatively homogeneous in amount and kind of mathematical experience its students had prior to the course, and the classes were assumed to represent a sufficiently differing spectrum of mathematical experience to reveal any systematic effects for mathematical experience. The instrument was administered at approximately the midpoint of each course and was not directly related to course material.

The instrument had four parts: (1) a brief background information questionnaire (gender, college major, etc.); (2) 40 bar graphs for which the subject was asked to rate on a seven point scale from "very typical" to "very non-typical" each bar graph as to typicality within the category of bar graphs; (3) ten graph reading and interpretation questions linked two each to five different bar graphs to re-focus subject attention indirectly on the ease of use of bar graphs of differing types; (4) a re-ordering of the same 40 bar graphs for which the subject was asked to rate on a seven point scale from "very easy to read" to "very hard to read" each bar graph as to its relative ease of reading within the category of bar graphs. Each set of 40 bar graphs differed on three dimensions — sub-type, orientation and pattern. Sub-type consisted of one of four types of bar graphs: (1) simple (single bars per level of some variable), (2) segmented (multiple shading for categories given in a legend, within a single bar per level of some variable), (3) grouped (multiple bars for categories given in a legend, for each level of some variable), and (4) isotypes ("bars" formed by the repeated use of some symbol, e.g., a square or triangle). [See Figure 2 for an example of each sub-type of stimulus.] Orientation was either horizontal or vertical (direction of bars). Pattern consisted of bars of increasing heighth (length), decreasing, symmetrically increase then decrease, symmetrically decrease than increase, and random.

Insert Figure 2 about here

Data gathered consisted of the response to each part of the instrument by each of the 98 subjects. Data was gathered during one class of the mathematics course in which they were enrolled. Data collection required approximately thirty minutes. All data was gathered by the same researcher, not the regular teacher of any of the classes.

Research Questions and Analytic Methods

The present investigation was limited to several research questions which could be approached through

the experiment just described. These include:

1. Are the typicality judgements of subjects such that they suggest that a concept or network of linked concepts exists in the mental representations of individuals and not just in a *priori* category judgements of the experimenters?
2. Do the typicality judgements reveal differences characteristic of groups with differing mathematical experience?
3. Are typicality and readability judgements linked to the complexity of the graphical perception task involved for the stimuli?
4. Are readability judgements linked to typicality judgement for the stimuli, or are readability and typicality equally "primitive" as features of the mental representations by which the graphical stimuli are rated?
5. Do the readability judgements reveal differences characteristic of groups with differing mathematical experience?

The data from each group for the typicality ratings was analyzed using PROC ALSCAL, available from but not supported by the SAS statistical package to provide a two-dimensional multidimensional scaling so that a two-dimensional "typicality space" could be graphed for each of the three groups. The same data was used as a basis for a cluster analysis using PROC CLUSTER of the SAS package. Each cluster was graphically indicated by contours in the typicality space, as were codes for bar-graph sub-type, orientation, and pattern. A similar graphical work-up was provided for the reading ease ratings for each group.

After these graphical displays were worked up for each of the two types of ratings and each of the three groups, visual inspection was used to identify patterns of significance in the features of the groupings that emerged. To confirm this more exploratory method, multiple regression and discriminant analysis techniques were used.

Results and Discussion

Typicality

Typicality judgements and effects are some of the most common tools used to study conceptual structure and representation in human cognitive processing and, in particular, to distinguish and refine differing theories of human conceptual representation (see, e.g., Medin, 1989). These typicality judgements have most often been gathered by asking subjects to list exemplars of a concept and examining frequency of mention (e.g., Rosch & Mervis, 1975). Also used are yes/no judgements of category inclusion and asking for lists of defining features of natural categories. The present instruments' use of "semantic differential" sort of rating followed by multidimensional scaling is less usual but is a technique that is very suitable for examining group differences in typicality structures. This investigation, however, should be supplemented by the more conventional approaches to conceptual structure and typicality effects.

Insert Figures 3, 4 and 5 about here

Figures 3, 4 and 5 present the "typicality spaces" for the three groups of differing mathematical experience. The cluster diameter represented by the contour lines in each figure is constant both within and across the three figures. The structure that appears shows typicality effects representative of a coherent conceptual structure in subjects' mental representations, since similar graphs were grouped together by the subjects. All groups rated simple bar graphs as most typical, grouped bar graphs next, followed by segmented bar graphs, with isotype bar graphs rated the least typical. Thus, in response to the first research question, the conceptual structures do seem to be reflected in subjects' mental representations and not represent sim-

ply a *priori* judgements of the experimenters — the close clustering of similar stimuli represent an internal representation of conceptual structure.

Additionally, there were clear differences in how subjects in the different levels of experience rated different graphical stimuli, shown by differences in distance between the stimuli in the configurations found. The lease experienced group revealed a fairly primitive structure with three clusters of broad heterogeneity. More experienced groups revealed tighter (more closely grouped, smaller proximity measures) clusters with more homogeneity per cluster than did the more experienced groups. That is, it is likely that the less experienced groups had knowledge representations consisting of models for several distinct sub-categories within the broad category of "bar graph" and that each sub-category was more stereotypic and homogeneous than was the case for the clusters of the least experienced group which had a more diverse array of graphs clustered within proximity clusters of the same size. (See Figure 6 for a sense of cluster tightness for the three groups. Further, less experienced groups appeared to use fewer stimulus features in clustering than did more experienced groups. Graph type and orientation seemed to be salient characteristics for grouping, but pattern was not.

Insert Figure 6 about here

Thus, in response to the second research question, individuals of differing mathematical experience have differing views of the typicality of bar graph stimuli and these differences show a characteristic pattern of increasing complexity of conceptual structure. Typicality seemed clearly to be a characteristic feature of the knowledge representations for "bar graphs" which helped characterize each group. Discriminant analyses to identify group membership on the basis of all or some of the graph ratings often had over 80 percent accuracy.

Readability

Coherent reading configurations were found (see Figures 7, 8 and 9). Simple bar graphs were rated the easiest to read with the isotype being rated the next most easy to read, followed by mixtures of both grouped and segmented bar graphs. The clusters for each group differed somewhat from experience group to experience group, but not as pronouncedly as did the findings for typicality.

Insert Figures 7, 8 and 9 about here

Two factors were found to be linked to the readability ratings for the graphs. The first was a measure of the complexity of each type of graph, based on the number of elementary graphical perception processes needed for each type of graph (see Cleveland, 1985). The coefficients of determination for the different groups for perceptual complexity predicting readability were $R^2 = 0.534$ for Intermediate Algebra, $R^2 = 0.620$ for Calculus II, and $R^2 = 0.508$ for Applied Statistics. That is, perceptual complexity (as number of elementary perceptual processes — length judgements, etc.) consistently explained over half of the variance in readability ratings for all groups. Clearly the answer to the third research question is "yes" — readability is linked to perceptual complexity.

The second factor related to readability was typicality measured as discussed above. This had a moderate effect on readability ratings. Coefficients of determination for the different courses were $R^2 = 0.278$ for Intermediate Algebra, $R^2 = 0.244$ for Calculus II, and $R^2 = 0.262$ for Applied Statistics. That is, typicality effects also explained a significant portion — about one fourth — of the variance in readability ratings, but consistently accounted for less than did numerical estimates of perceptual complexity. While these two factors — perceptual complexity and typicality — did explain most of the variance in readability ratings,

there was still sufficient variance unexplained to make it unlikely to be due simply to random error or to individual variations without systematic base. This is a subject for further research.

In answer to the fourth research question, however, readability is clearly linked to typicality. In contrast, typicality was found to depend much less on readability and on perceptual complexity virtually not at all. This suggests that typicality is a more primitive characteristic of individual stimuli in relation to the mental models used to interpret and comprehend bar graphs. Readability seems less primitive, depending significantly on at least two other factors, one of which (complexity) is a perceptual rather than a memory feature. This is at least a preliminary answer to the fifth research question.

Concluding Remarks and Educational Significance

The results of this study have confirmed that both memory processes (typicality as an indicator of internal conceptual structure) and perceptual processes (perceptual complexity) do affect the views of individuals of how easy or hard a graph is to read and hence understand. Other parts of the modified Pinker model need to be investigated for a better knowledge of how graphs are understood. Further, other probes are needed to more clearly explicate the nature of the mental models involved.

Furthermore, the memory processes, particularly, are affected by experience, even indirect experience, such as education in mathematics. Research on how this occurs should be undertaken and, in particular, effort should go into determining which correlates (if any) of mathematical experience make the difference.

Another topic for further examination is the relation of gender to graph comprehension. It was found on the reading task (the third part of the instrument) that there was an interaction between gender and level of experience in how well subjects were able to interpret graphs. An expanded version of this part of the instrument, with a research design balanced for gender numbers and with reliability and validity checked, would be useful to determine if gender is linked to differences in graph comprehension, and, if so, the degree and nature of this effect.

In terms of significance for educational research, the techniques used here to explore knowledge representations for naturally- or incidentally-formed mental models seem clearly to have demonstrated their effectiveness and their potential usefulness for further investigation. The present experiment would also seem to be a good start of investigation of knowledge representations that can be used to link research on perception of graphical data to research on interpretation and question-answering.

The findings of even this initial experiment seem to be educationally and instructionally significant. In efforts at enhancing quantitative literacy, it would be an easy mistake to assume that virtually all students or program participants had very similar underlying mental representations used in understanding quantitative — in this case, graphical — information and thus to underestimate the diversity of the populations which quantitative literacy programs must address and to do so in ways likely to make interventions less effective. A more accurate research portrait of common knowledge representations should serve as a more adequate basis for designing more efficient interventions. For example, based on the findings here, it would seem reasonable to address explicitly in graph interpretation materials the significance of orientation of the graphs being interpreted (or, rather, the lack of interpretational relevance). Without trying to link directly aspects of mathematical experience to differences in representation, it seems clear from the results here that differing experiences do lead to differing knowledge representations and that, given the social importance of such skills as critical interpretation of graphically presented data for an educated citizenry, these differing experiences may need to be compensated for in programs of deliberate intervention for some persons.

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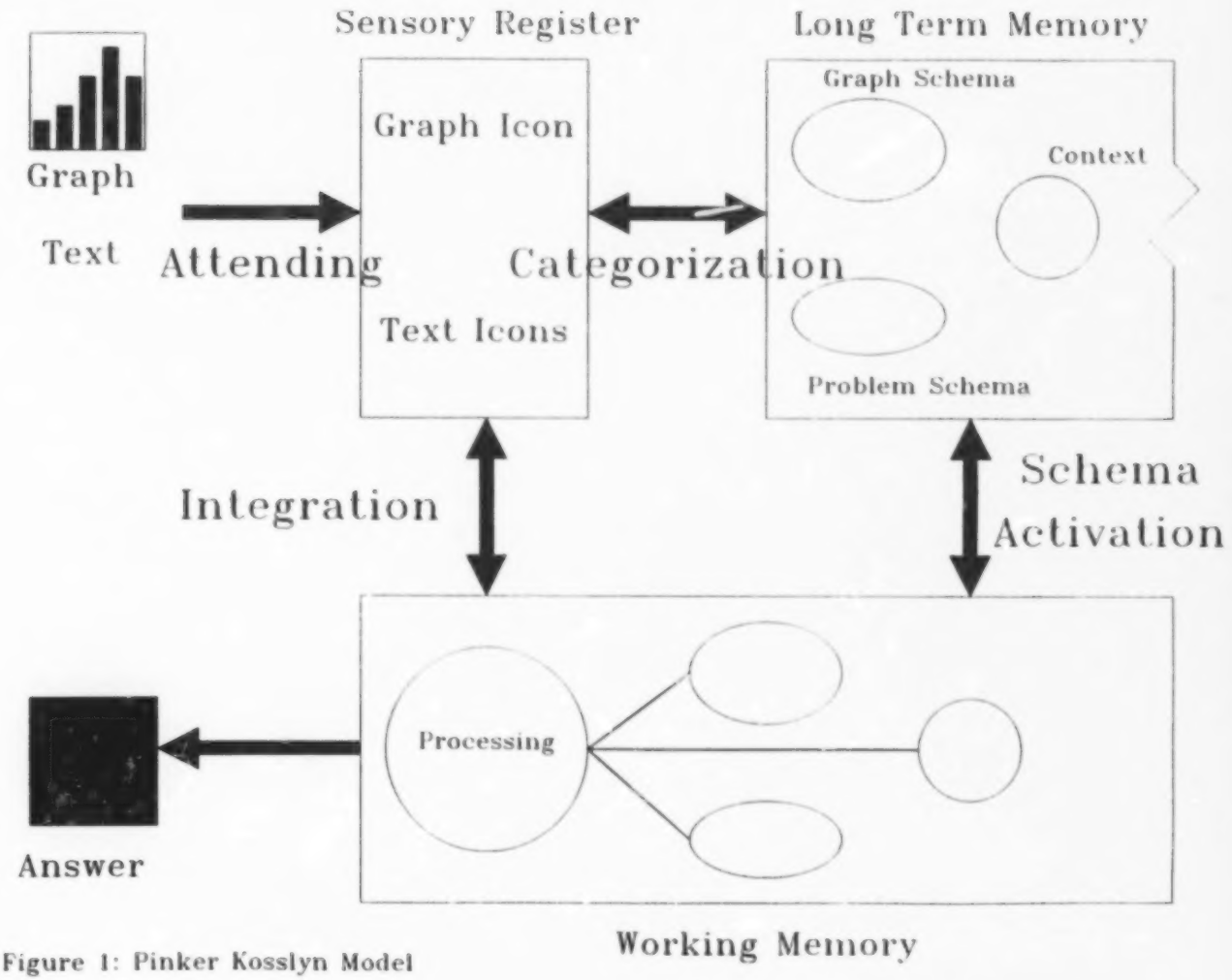
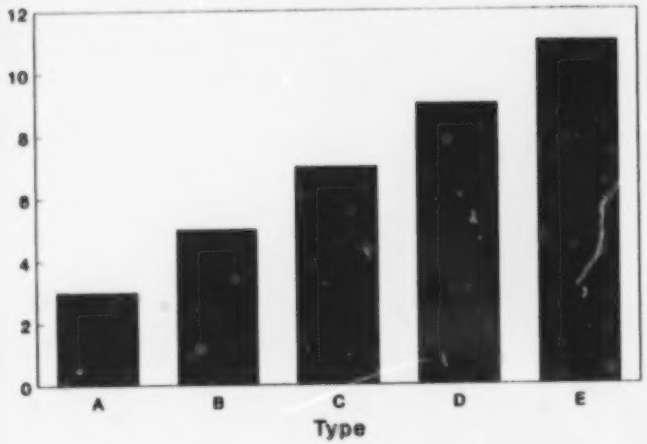
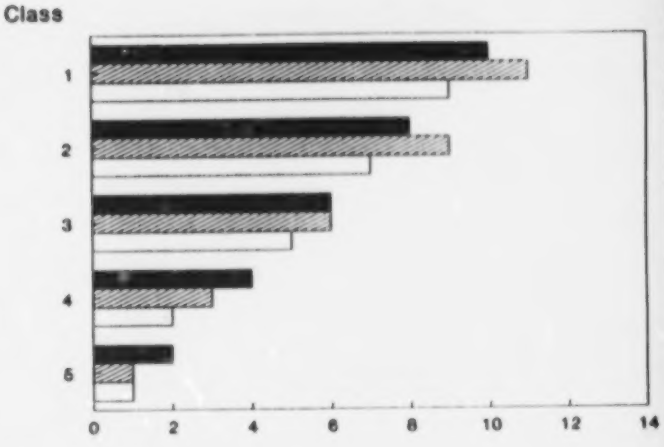
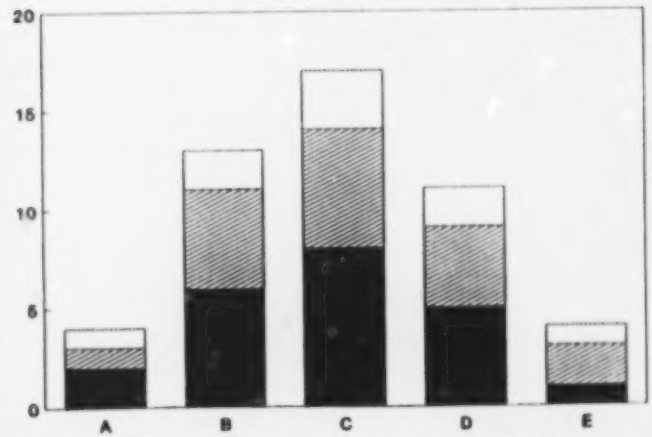


Figure 1: Pinker Kosslyn Model







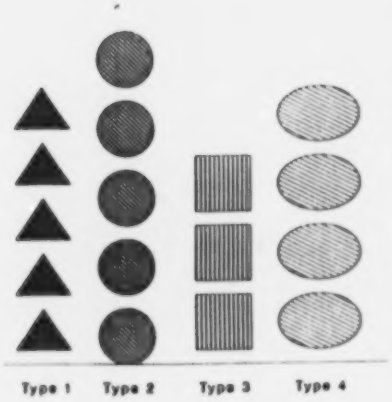


Figure 2: Examples of bar graph subtypes

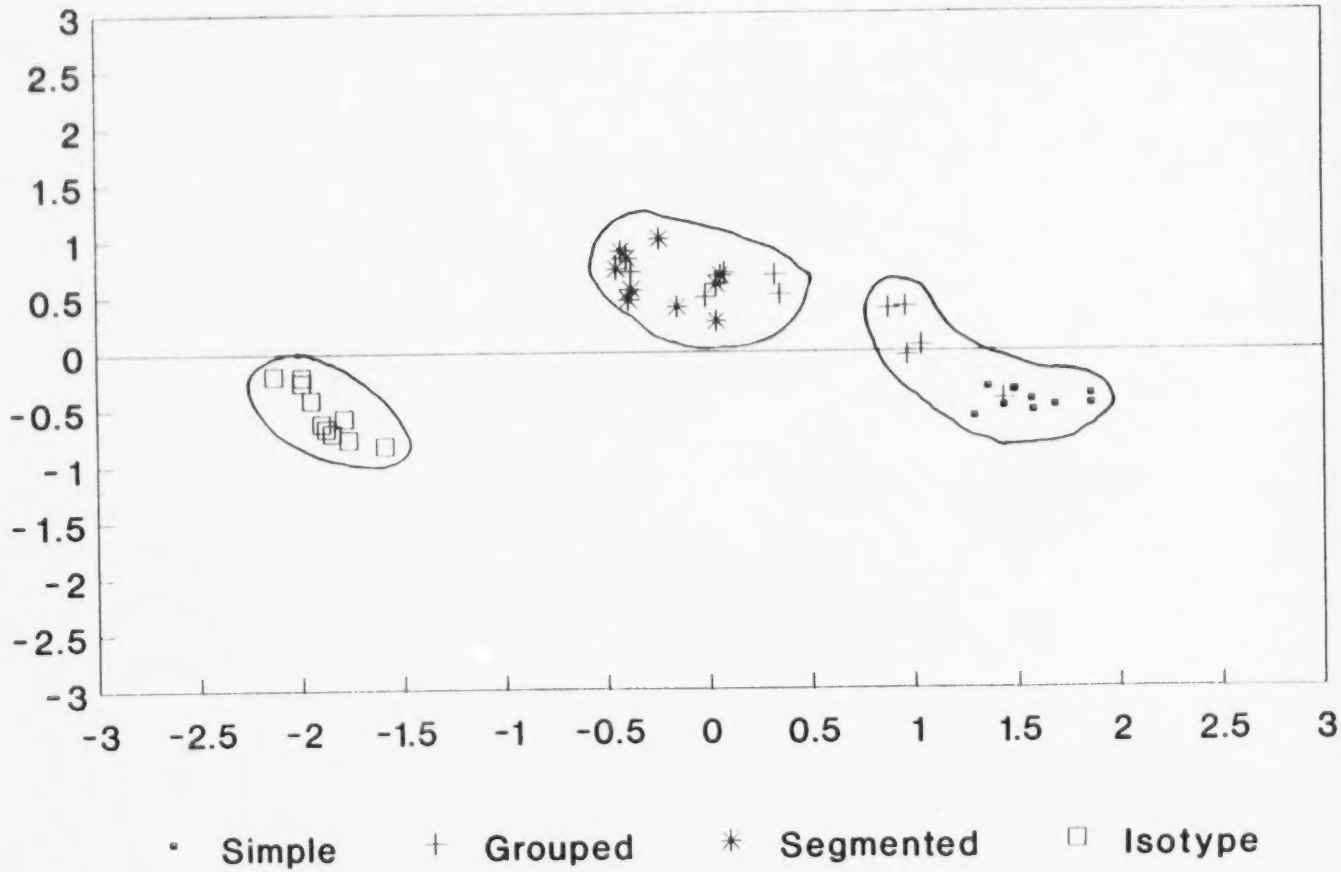


Figure 3: Typicality for Intermediate Algebra

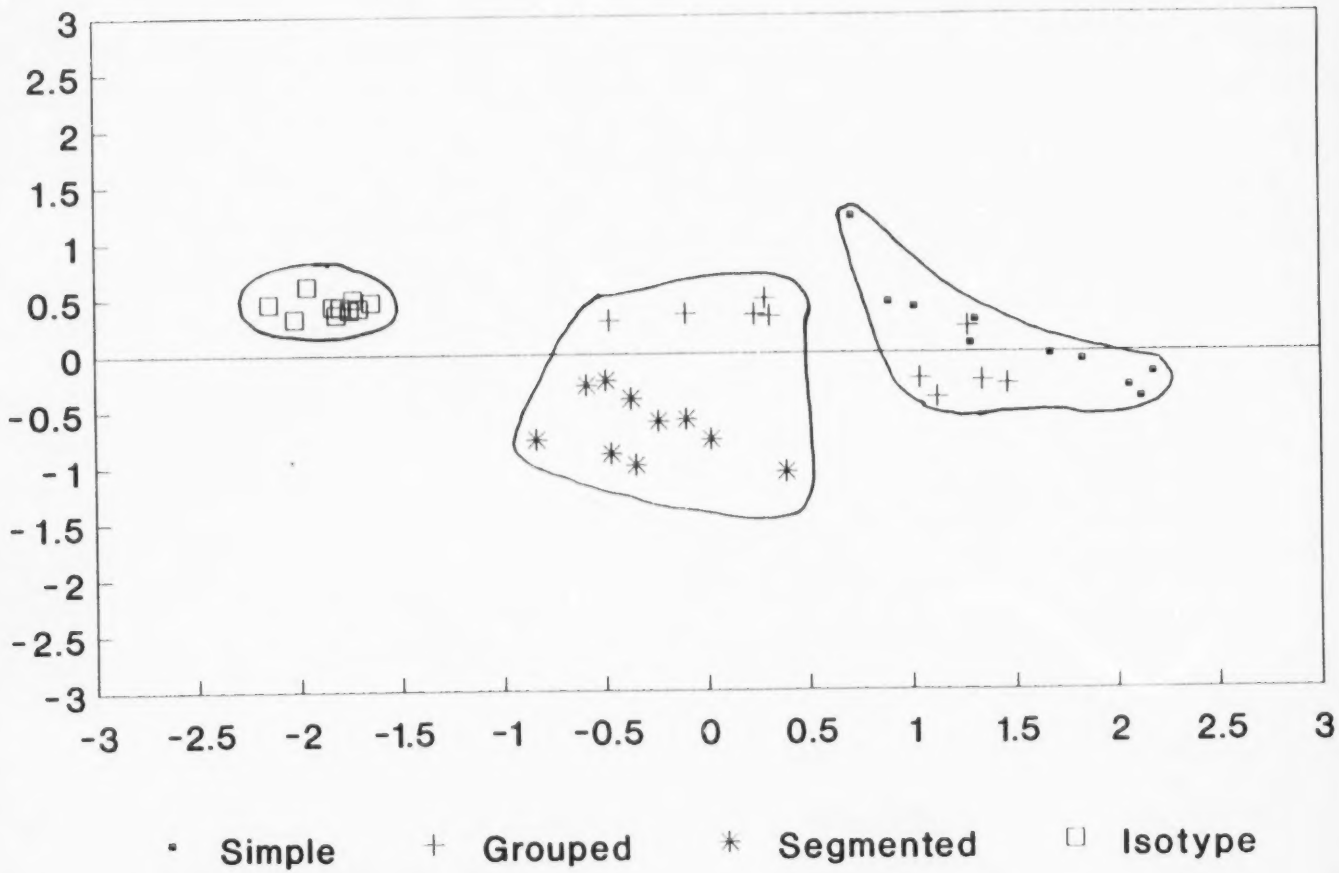


Figure 4: Typicality for Calculus II

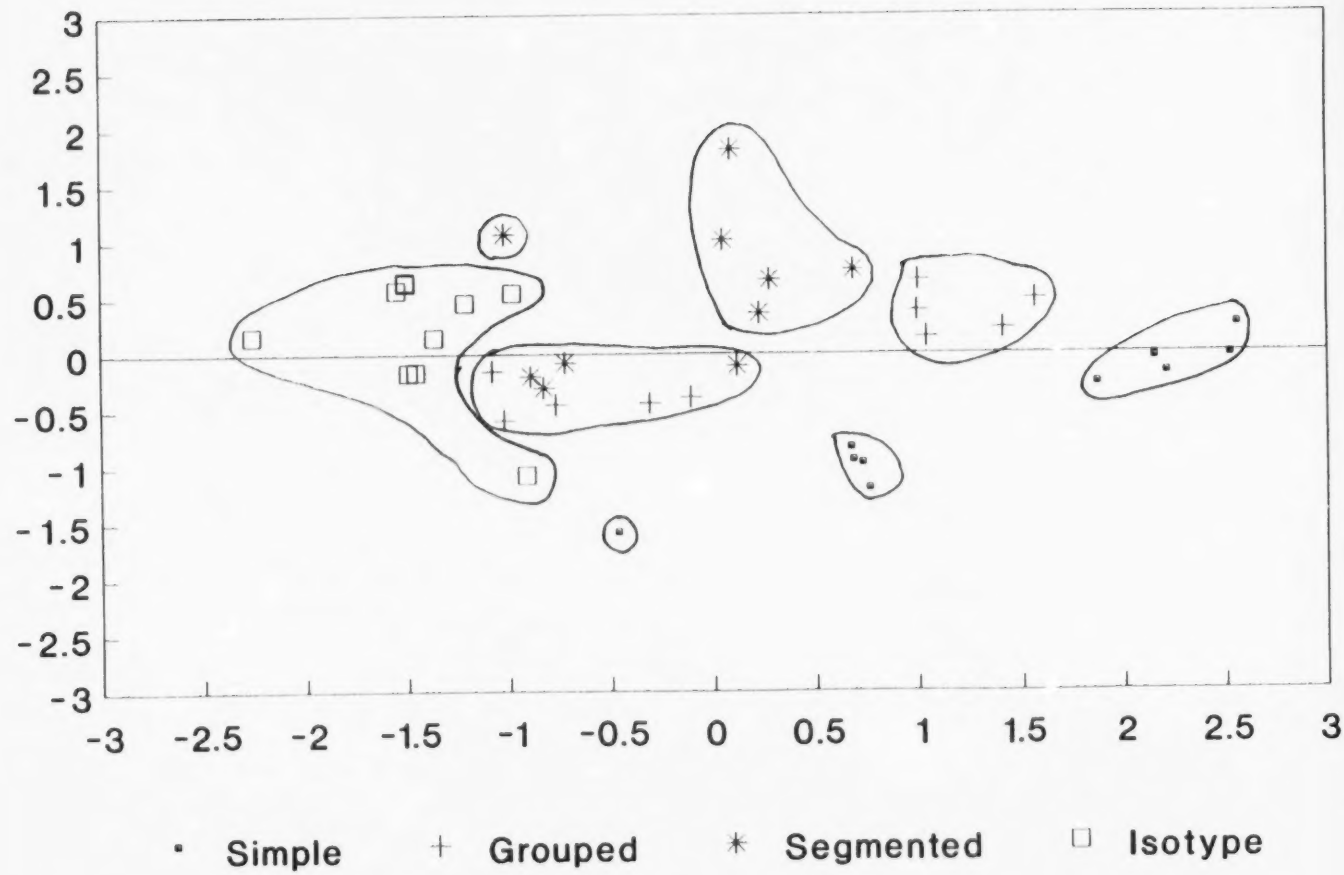


Figure 5: Typicality for Statistics

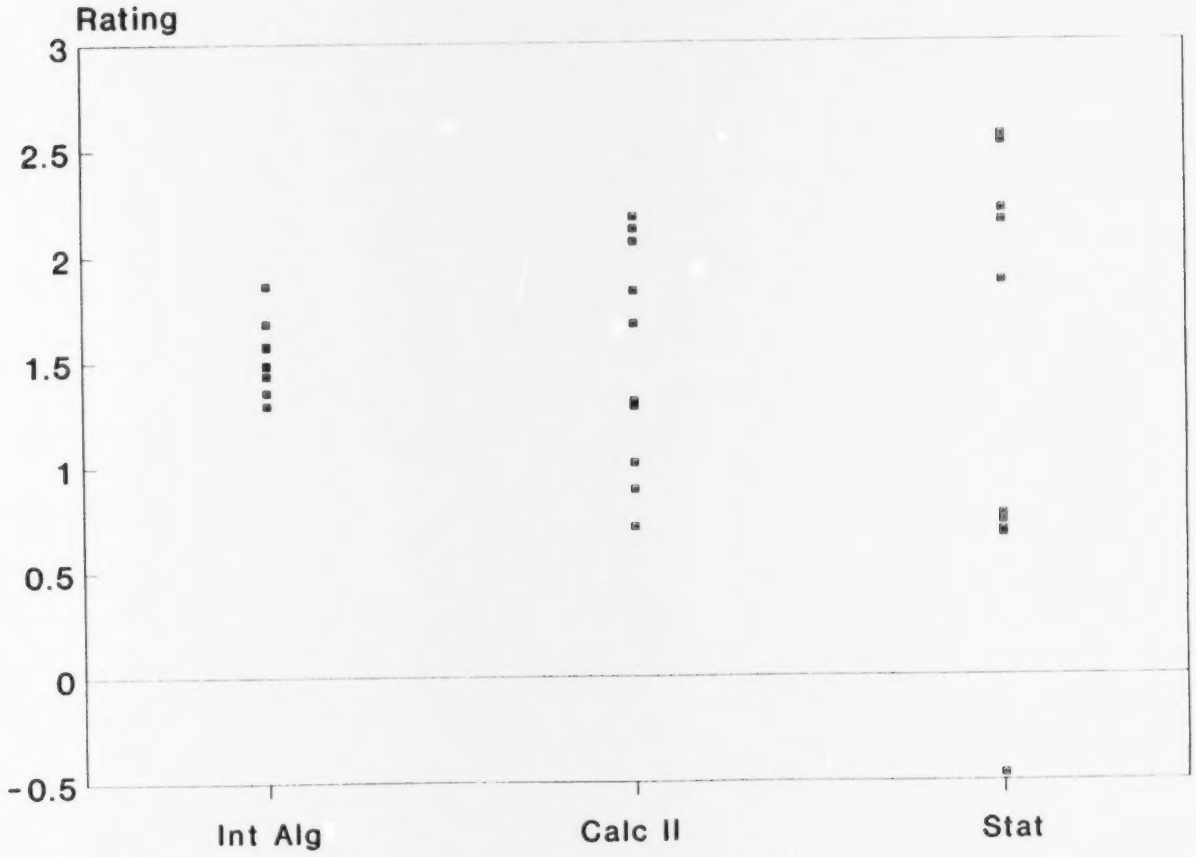


Figure 6: Typicality for Simple Bar Graphs

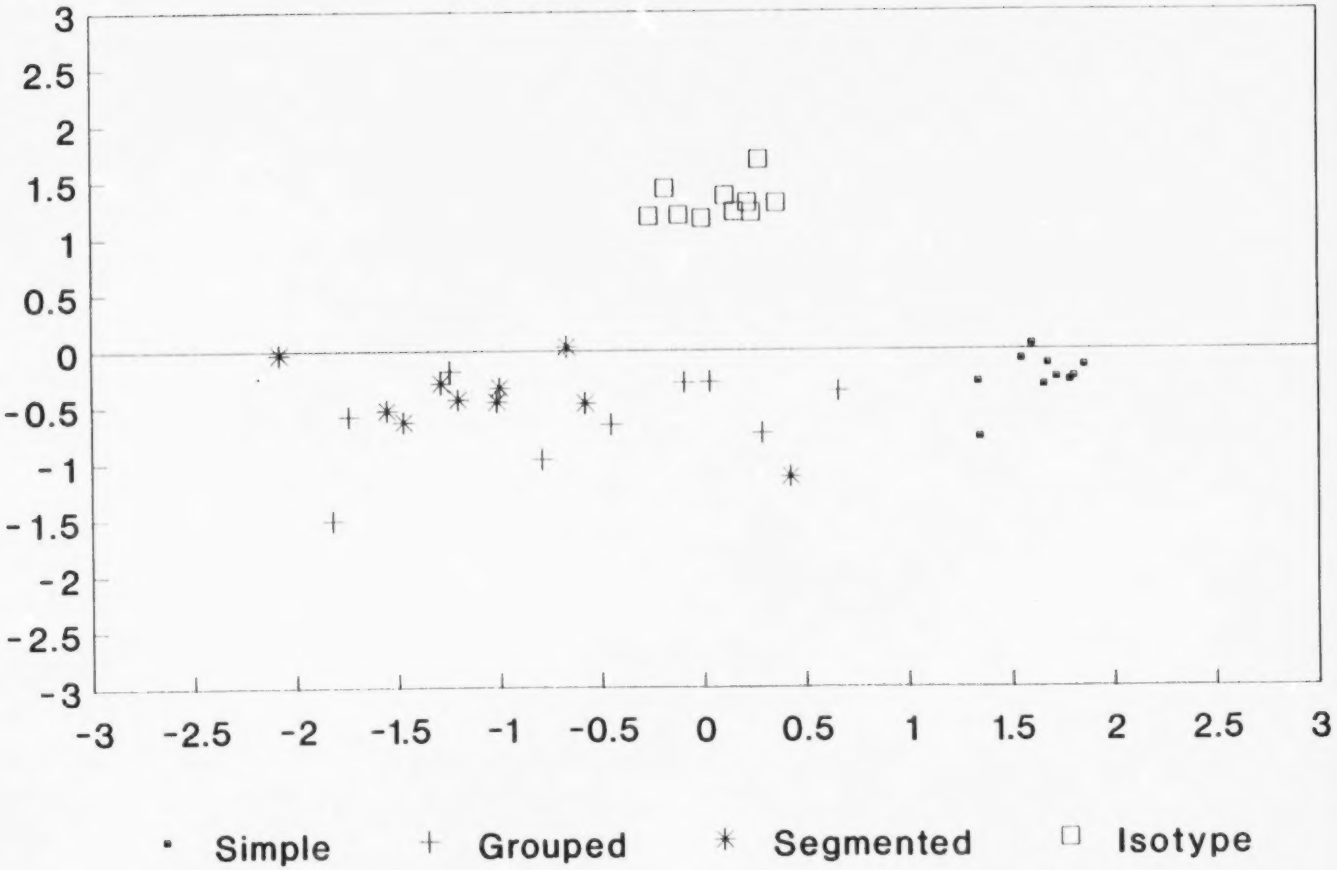


Figure 7: Readability for Intermediate Algebra

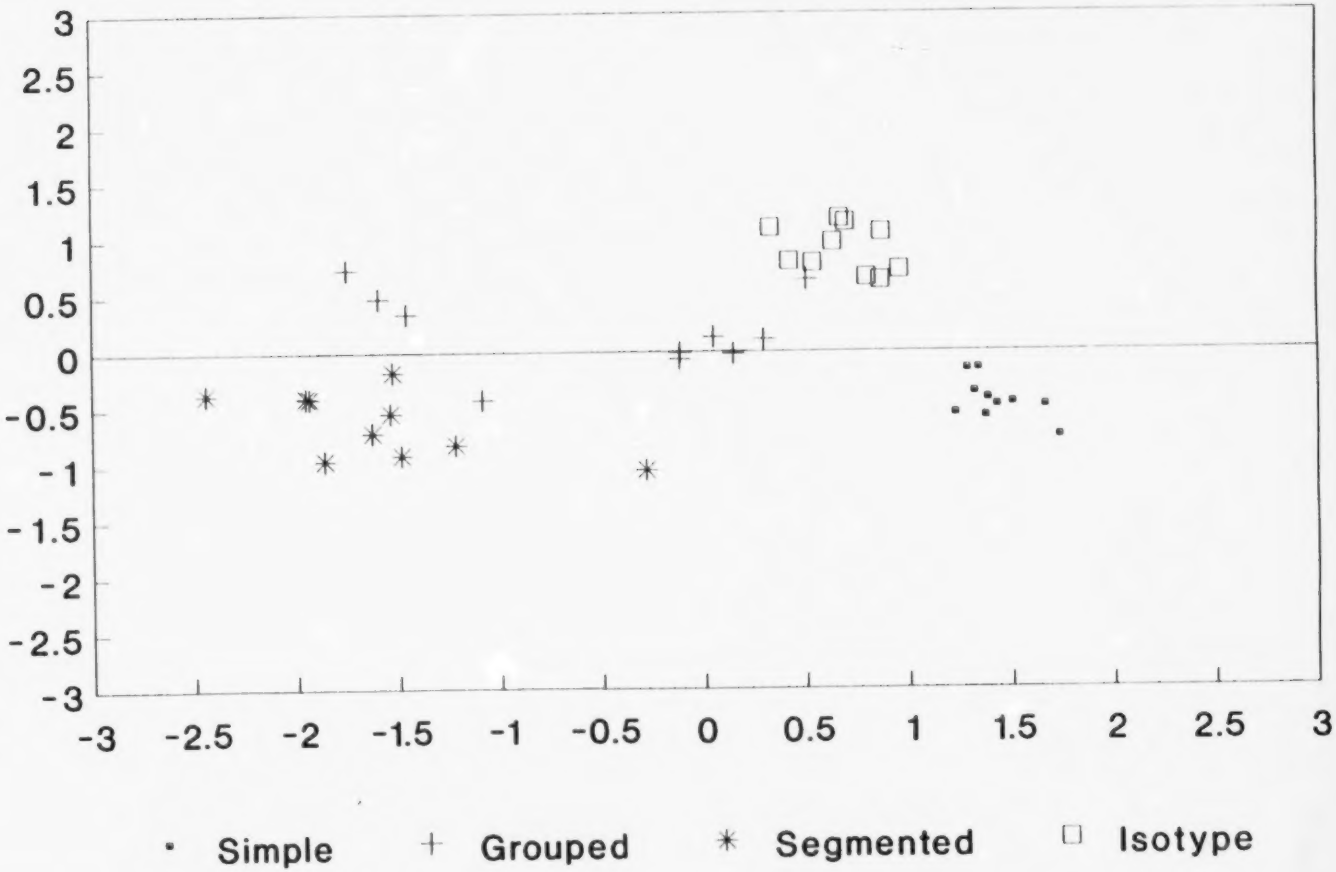


Figure 8: Readability for Calculus II

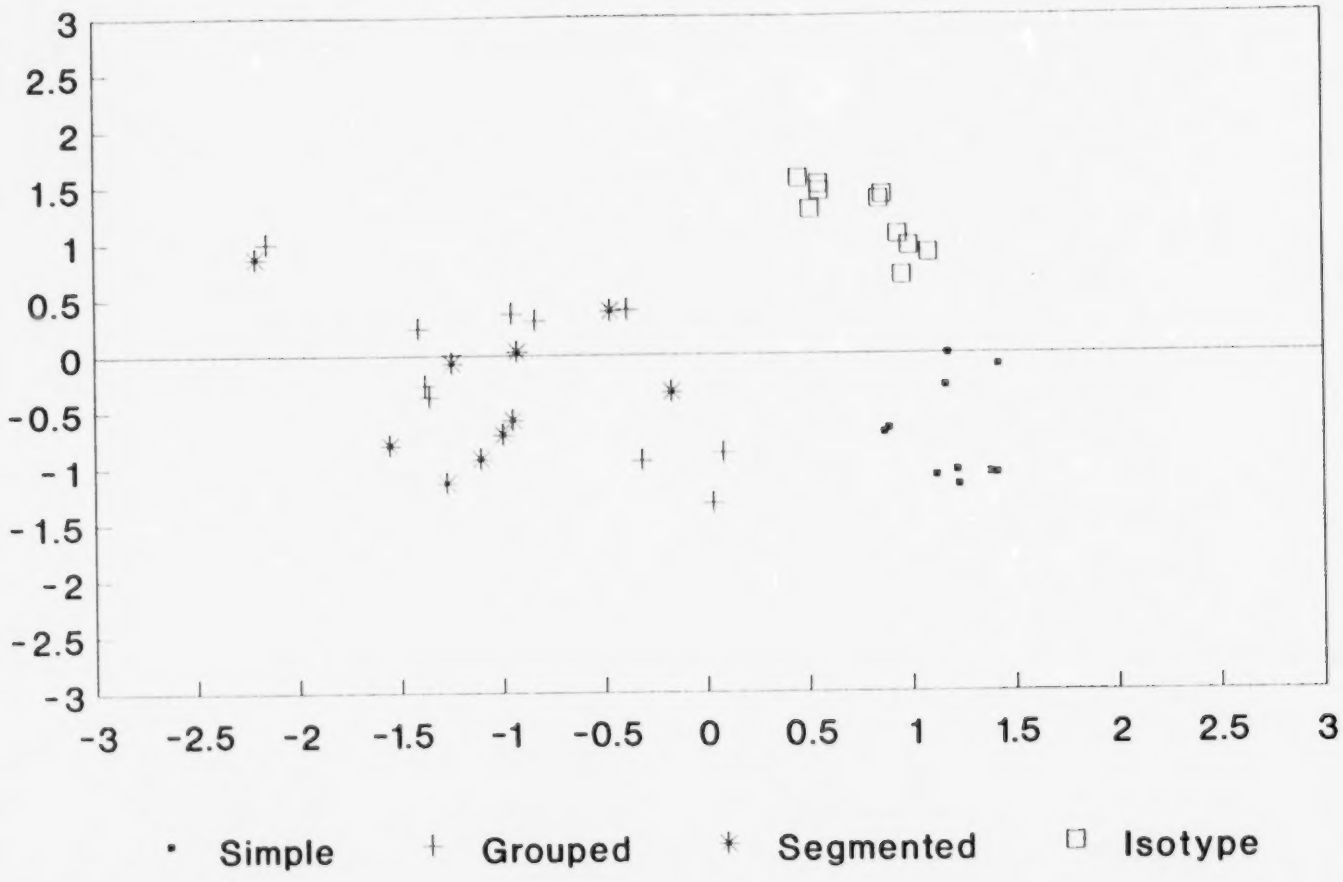


Figure 9: Readability for Statistics